CURRICULUM, PEDAGOGY AND BEYOND

THE MATHEMATICAL ASSOCIATION OF VICTORIA

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Giving a Guernsey to Geometry

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and Ms Connie Galati

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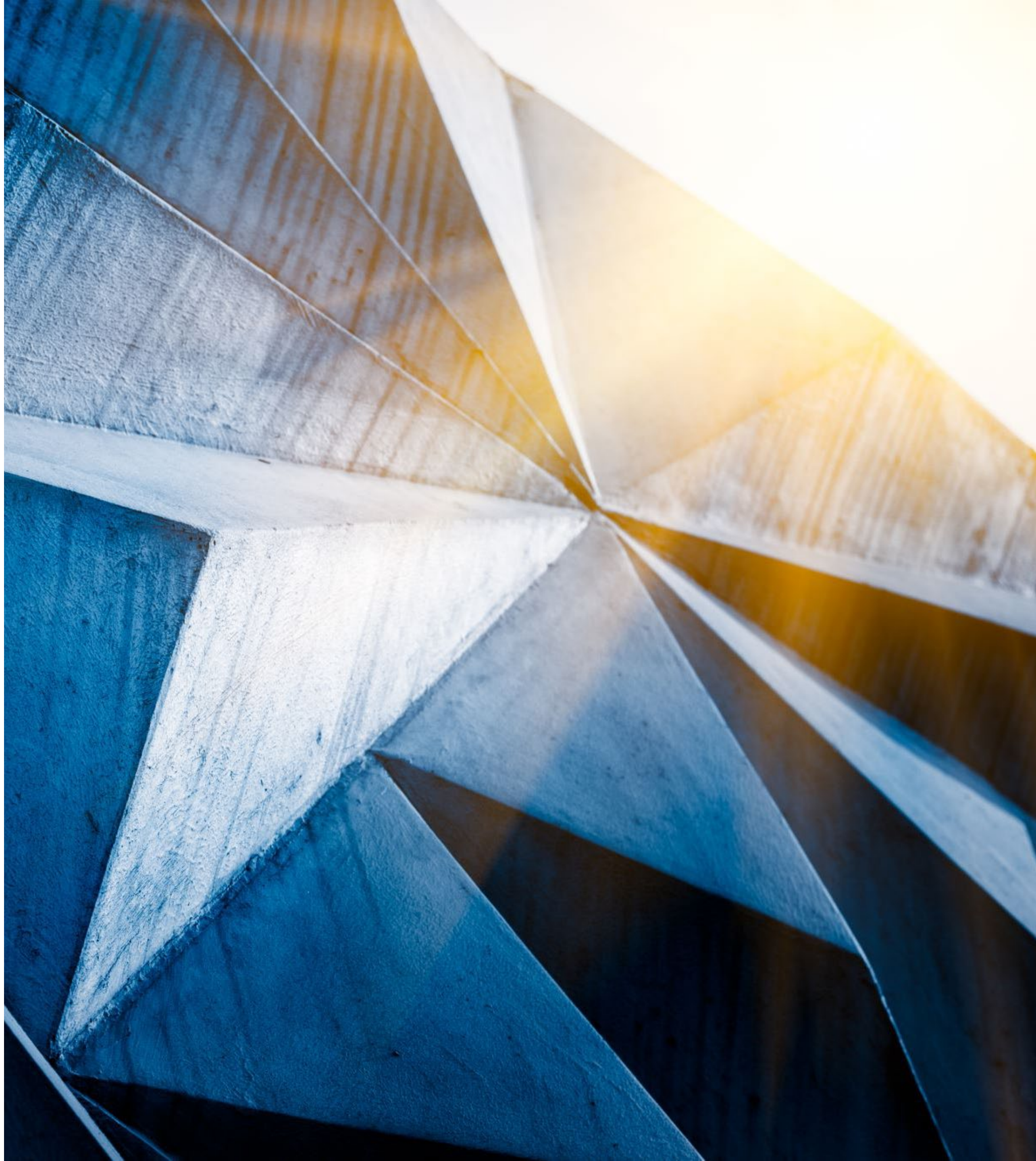
Acknowledgement of Country

- In continuing commitment to Reconciliation and in line with Aboriginal and Torres Strait Islander tradition, it is customary to acknowledge country as we pass through. Today, we acknowledge and pay our respects to the First Peoples, traditional custodians of the lands and waters and thank them for their continued hospitality. I acknowledge the Wurundjeri people whose land I am currently on and all the lands you are on today.
- We acknowledge and celebrate the continuation of a living culture that has a unique role in this region. We also acknowledge Elders past and present as well as our emerging leaders of tomorrow and thank them for their wisdom and guidance as we walk in their footsteps.

Learning Intentions

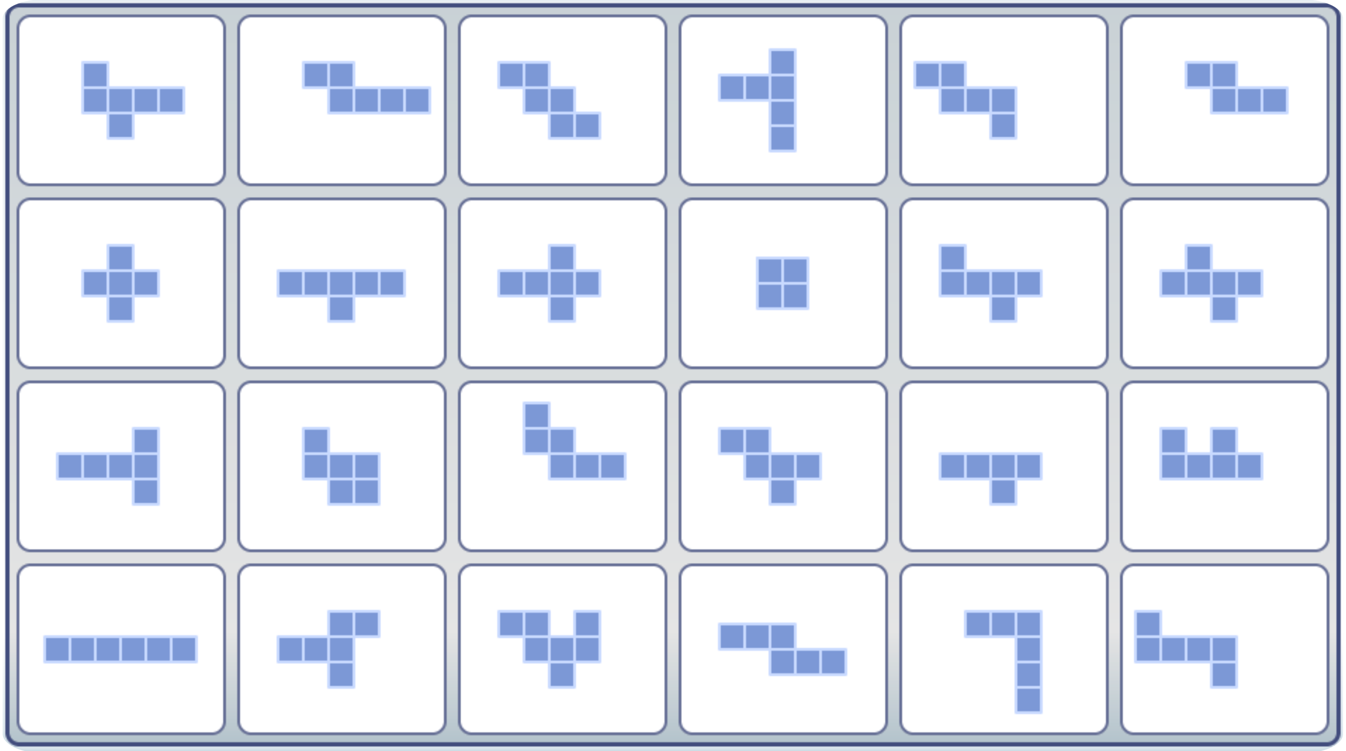
- To identify and describe big ideas and ways of working mathematically with geometry that students in Years 5-8 are required to come to understand and explain how these are captured in the Victorian Curriculum.
- To articulate and describe pedagogical practices, guided by the concrete representational abstract (CRA) approach, that have most impact on students's geometric reasoning including use of multiple examples and non-examples; use of imagery beyond prototypes; use of questioning and concrete/virtual manipulatives; and, encouraging representations and visualisations.



Why give a guernsey to Geometry?

- **Geometry is as important as algebra** as a basis for calculus.
- In geometry, students can identify the attributes of shapes and objects and how they can be combined or transformed. They are able to use **spatial reasoning, visualization** and geometric properties to solve problems which is important for a range of tasks.
- More general mathematical reasoning is involved many geometrical tasks.
- Geometry is applicable to Art, Architecture, Surveying and Building, Design, Astronomy and Aerodynamics.
- So why do teachers often leave it to the last topic for the year when students are more often distracted?

Warm Up Activity



Is Akol’s conjecture true? How do you know?

You might like to try this electronic version https://www.nctm.org/classroom-resources/illuminations/interactives/cube-nets/

Nets of a cube: Akol’s conjecture

After doing my investigating, 12 of those are nets for a cube.

What strategy will you u

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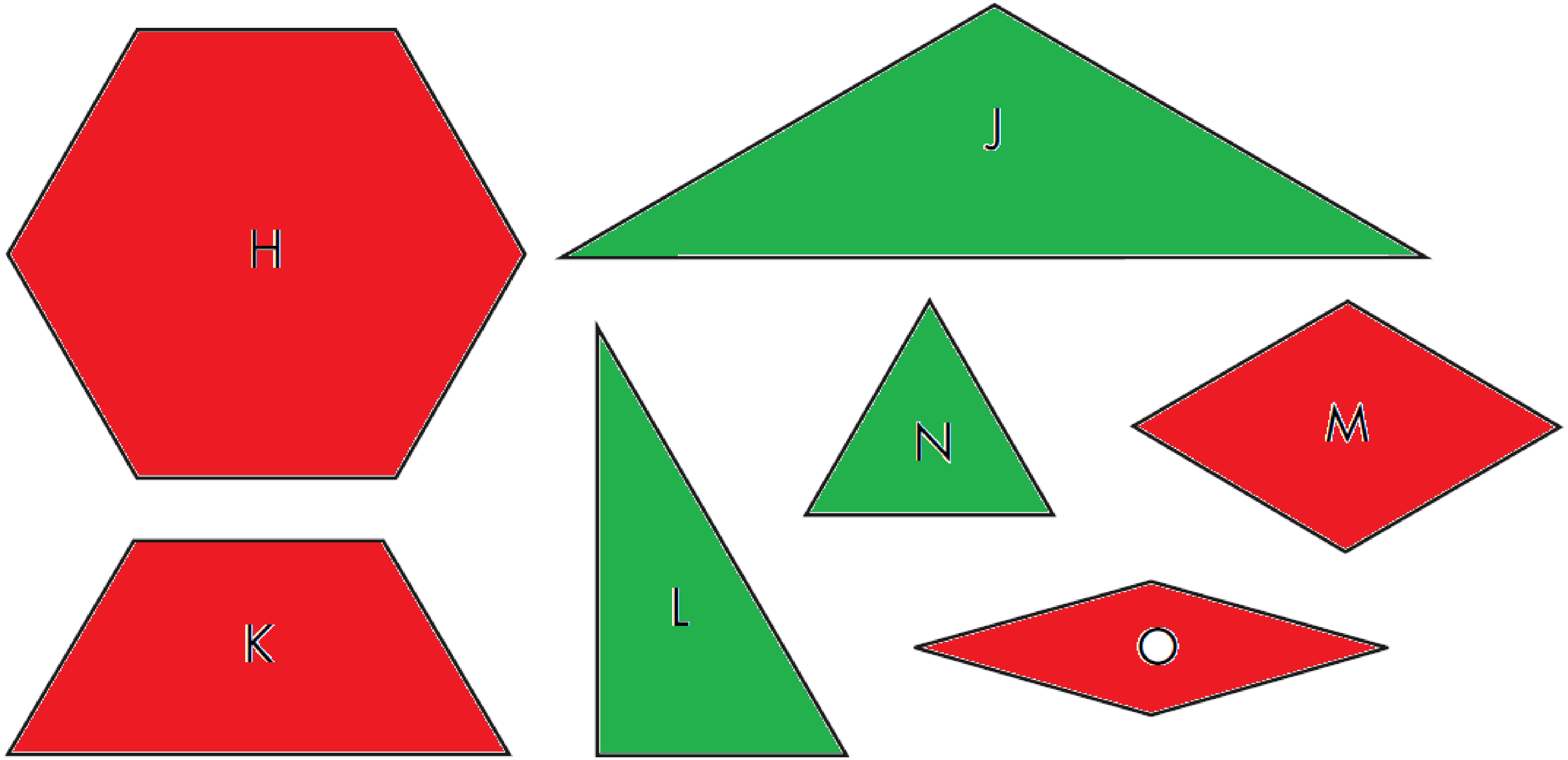
Two Dimensional Geometry



Key understandings

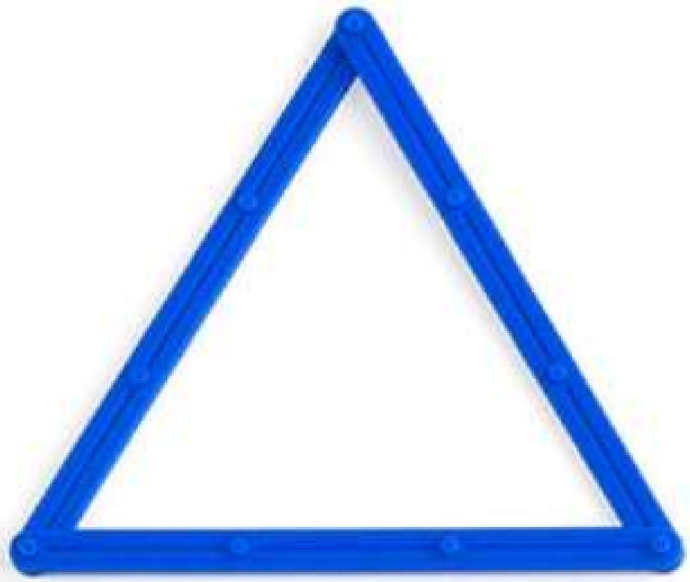
- Space and shape are often the first means through which children engage mathematically with the world.
- Geometric thinking includes different forms including spatial orientation, spatial reasoning, geometric reasoning, and spatial visualisation.
- Geometric understanding begins with the exploration of line and is followed by teaching that focuses on the importance of ‘closed’.
- Pedagogical and curriculum documentation is built on the ‘big ideas’ of mathematics. These big ideas are often implicit and therefore require teachers to possess deep mathematical content knowledge (MCK).
- For 2D geometry, there are important ‘big ideas’ that we want children to come to understand through our purposeful planning, teaching, and assessment practices.
- There are geometry-specific teaching strategies that effective mathematics teachers use to support children’s learning including the way we name figures by focusing on properties, using many examples that go beyond prototypical images, and holding critical mindsets about commercially produced resources.
- Effective teachers are mindful of displays, ensuring that multiple examples are used (that go beyond prototypical images) as well as using non-examples. These are used to engage children in dialogue as a means of deepening their geometric thinking.

Geo-stick play

Close your eyes

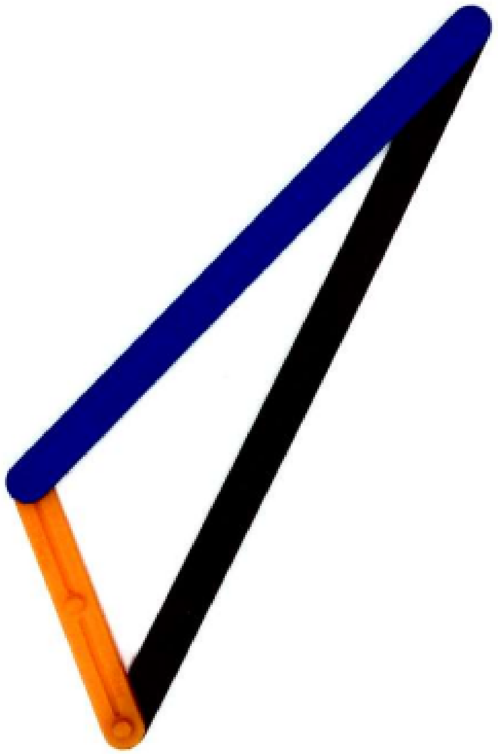
Visualise a triangle

Make the triangle you visualised using the <u>minimum number</u> of Geo-Stix



Did you represent the standard, **prototypical** triangle?
- standing on its base
- equilateral or isosceles?

My triangle

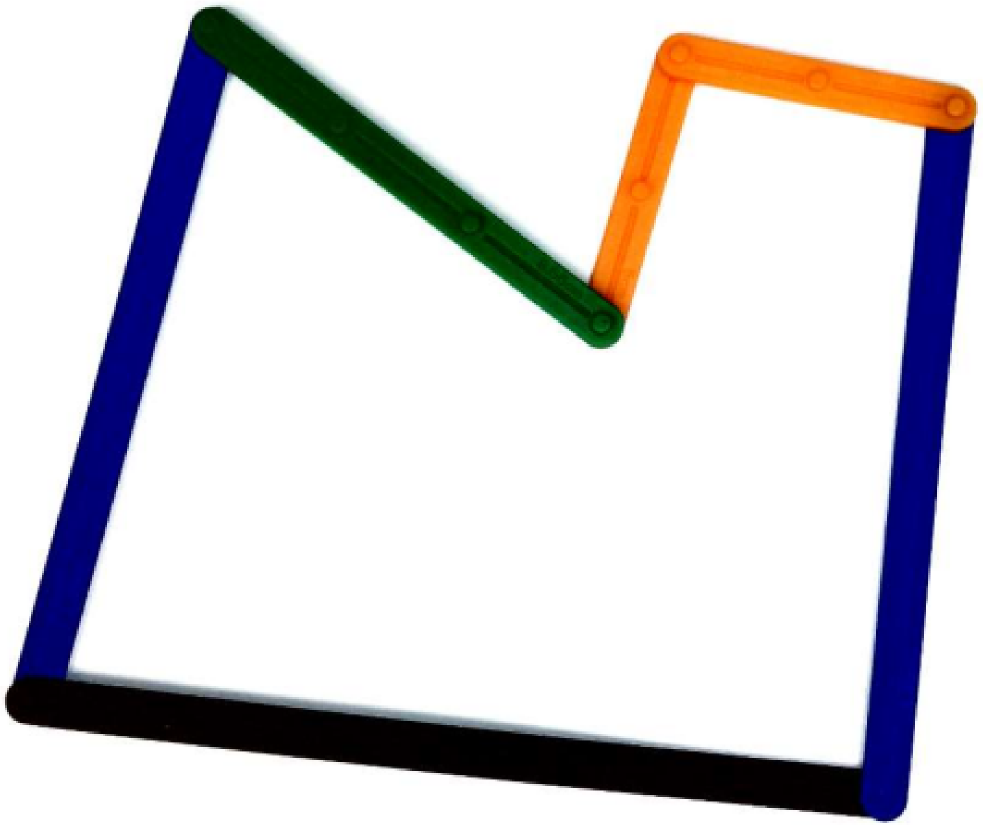


Close your eyes

Visualise a hexagon

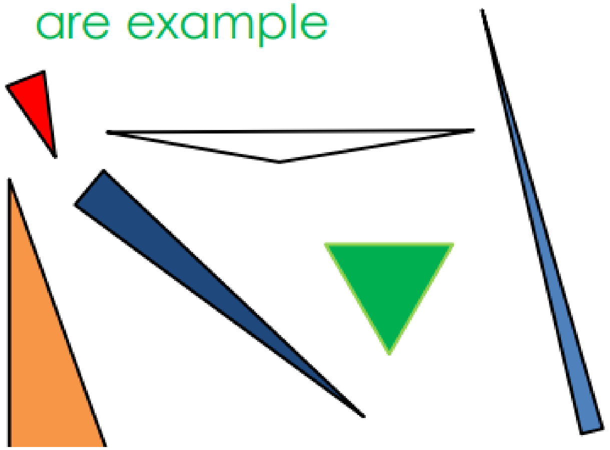
Make the hexagon you visualised using the <u>minimum number</u> of Geo-Stix

My hexagon

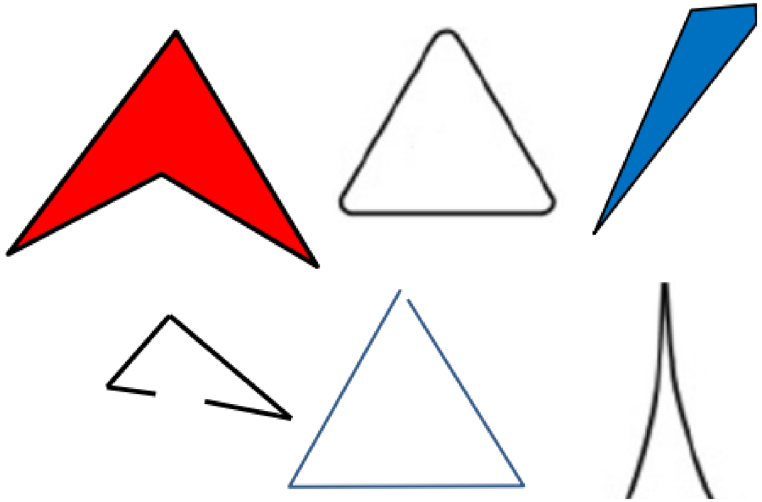


Power of many examples and non-examples

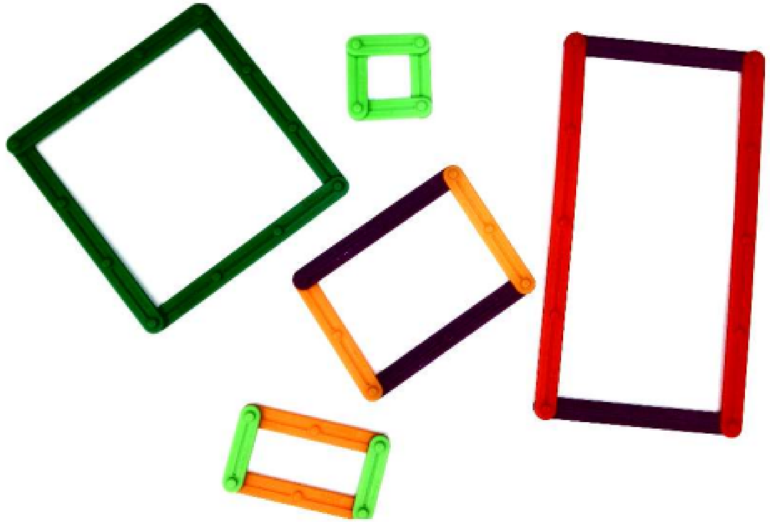
Many **examples** going beyond prototypes, and explaining why indeed these are example



Many **non-examples**, and explaining why this is the case



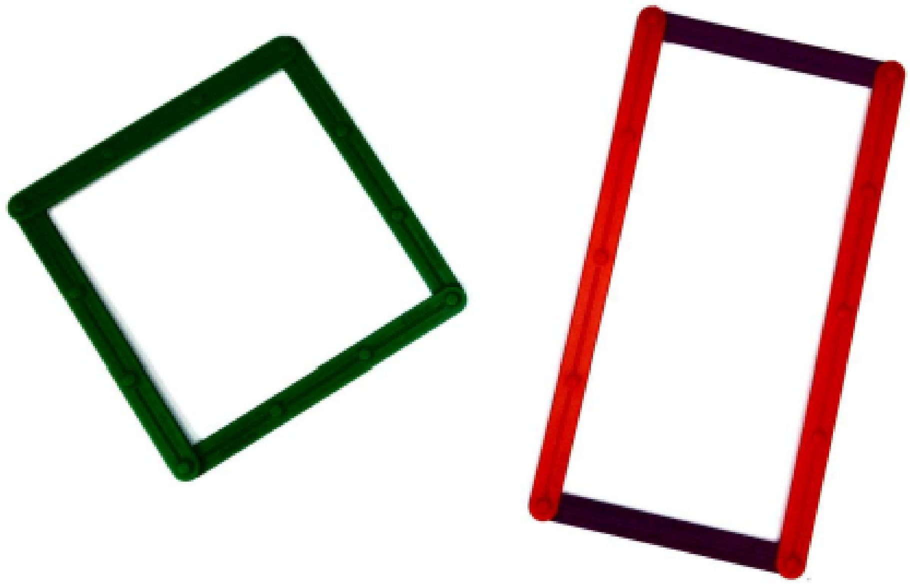
Classifying



I put these Geo-Stix 'figures' into a group.

What was my 'sorting rule'?

Focus more on similarities



How are these two figures the same?

What properties do they share?

By focusing more on the similarities (the shared properties), we can develop deeper understandings of geometry.

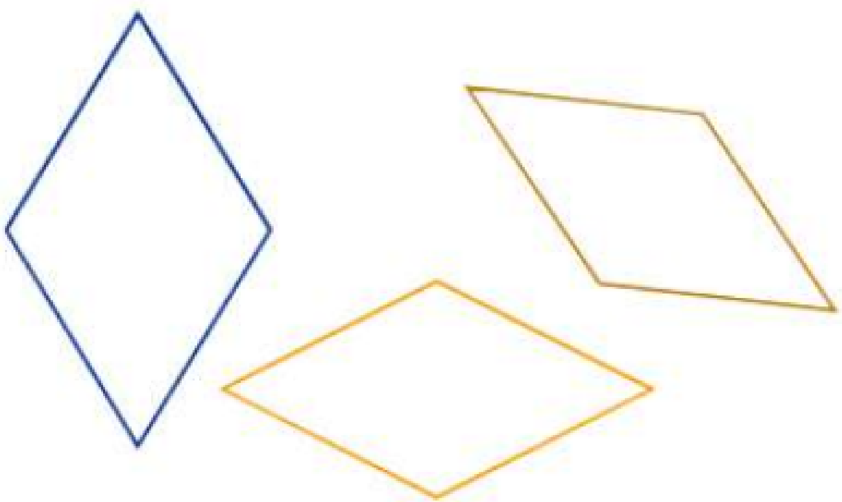
Why are these questions vital for children in developing their geometric reasoning?

Impact of looking for similarities

When we ask children to look not only for differences but also similarities, this can help children to:

- Focus on properties as important features of figures (shapes)
- Notice relationships between classes of figures
- Build understanding of hierarchical thinking (reasoning that squares are special cases of rectangles and rhombuses)
- Move to more sophisticated levels of thinking as articulated in the Van Hiele Levels of Geometric Reasoning (abstract/relational level)

Rhombus (not a diamond)



A rhombus is a **parallelogram with all sides equal (congruent) in length.**

We also do not use the term “diamond” for a rhombus.
In everyday use, a diamond is a precious jewel, the name for a baseball pitch, or one of the card suits.

Three dimensional Geometry

Important ‘big ideas’ of 3D geometry

- **Properties**
  - **3D solids** (edge, face, vertex, angle, diagonals, concavity, symmetry incl reflective & rotational)
- **Classification (including hierarchical thinking)**
- **Representation**
- **Transformations (isometric and non-isometric)**
- **Visualisation**
- **Relationships between 2-D figures and 3-D solids**

Key understandings

- The important ‘big ideas’ of 2D geometry are so 'big' that they apply to 3D geometry. Helping children see how those big ideas apply to both plane (2D) and solid (3D) geometry supports their conceptual understanding of geometry.
- The Van Hiele 'Levels of Geometric Reasoning' is an important framework that teachers can use to support their planning, teaching, and assessment of and for children's geometry learning. The framework can be used for 2D and 3D geometry.
- Effective teachers understand the characteristics of thinking at each level of the Van Hiele framework (knowing that there are two other levels of thinking that are 'Formal deduction' and 'Rigour' levels that secondary students learn) and choose appropriately challenging learning experiences/tasks that support the transition from one level to the next.
- Children's transitions in geometric reasoning take extended periods of time and require many, varied experiences that allow children to connect with important big ideas and provide opportunities to enact the geometric reasoning at each level.
- The pedagogical approach offered by the Van Hiele's can support geometry instruction. The phases of Information, Guided Orientation, Explication, Free Orientation, and Integration allow for teachers and children to engage in geometry experiences with the purpose of extending children's geometric reasoning, conceptual understanding, and ways of working mathematically with geometry.

TABLE 1

The Van Hiele theory of geometric thought describes the different levels of understanding through which students progress when learning geometry.

Van Hiele theory of geometric thought

Level	Description	Ability of student
1	Visual	Describes shapes on the basis of their appearance
2	Analysis	Describes shapes on the basis of their properties
3	Abstraction	Recognizes the importance of properties and the relationships among them, which assist students in logically ordering the properties of the shapes
4	Deduction	Attains logical reasoning ability and proves theorems deductively
5	Rigor	Establishes and analyzes theorems in different postulation systems

Usually from Foundation to Year 1 or Year 2

It's a square because it looks like a square

Usually from Year 3 to Year 5

Squares have 4 sides that are equal, 4 right angles and parallel sides

Usually from Year 6 to Year 9

All squares are rectangles but not all rectangles are squares.

Van Heile’s model – assessing levels

- The Van Hiele theory is based, in part on the notion that student growth in Geometry takes place in terms of identifiable levels of understanding and that instruction is most successful if it is directed at the student's level
- Students cannot reach a higher level without passing through the lower levels.
- What is important at one level is subsumed by perceptions at a higher level.
- If we could find out just how much (or how little!) our students understand geometry, then we would be able to teach them more effectively.
- Rote learnt descriptions appear to be of little value to either the student or the teacher. In fact, having students memorise rules can cloud over the real level of understanding of the student and allow teachers to wrongly identify the true needs of students.

ACTIVITY 1. (Descriptions)

- Students are asked initially to draw a given figure.
- Suitable figures would be a square, rectangle, parallelogram and rhombus.
- This will help the teacher to be aware of whether the figure the teacher is talking about is the same as the one the student has considered.
- The students are then asked to write down a description of this figure ‘as they would to a friend over the telephone'. The question should be repeated for each figure.

ACTIVITY 2. (Minimum properties)

- Students are asked to write new descriptions of the figures described in Activity 1 using the least or smallest number of 'things' (i.e., properties) that would still allow the figures to be described or identified.

ACTIVITY 3. (Class inclusion)

- Students are asked to describe various figures in terms of another figure. An example would be to describe a 'square' but to use the word 'rectangle' in the sentence description.

Understanding Geometric levels of…

JOHN PEGG - University of New England

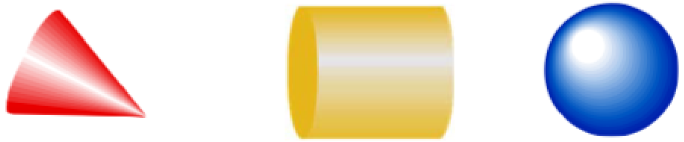
GEOFF DAVEY - Griffith University

The Australian mathematics teacher vol 47 no 2 1991

The importance of correct terminology

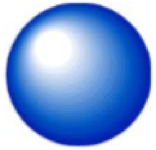
Solids

- Three-dimensional (3D) shapes are called ‘solids’ or ‘objects’
- Among the simplest solids are the sphere, cube, cone, cylinder, and more generally, the **polyhedra**.

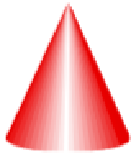


Types of solids

Sphere: one curved surface where every point on that surface is the same distance from the sphere's centre

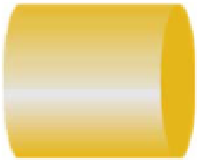


Cone: consists of a circular base, a vertex (corner) in a different plane, and line segments joining all the points on the circle to the vertex



Types of solids

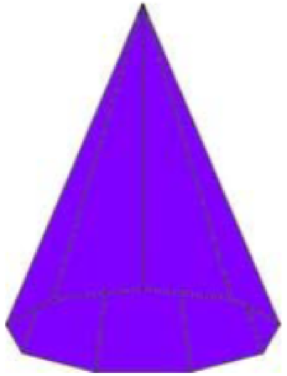
Cylinder: object whose uniform cross-section is a circle; a right cylinder can be defined as having circular bases with a curved surface joining them

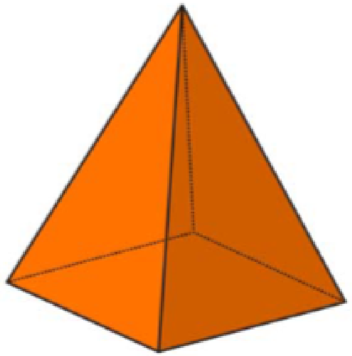


Cube (hexahedron): a rectangular prism with six congruent square faces

Types of solids

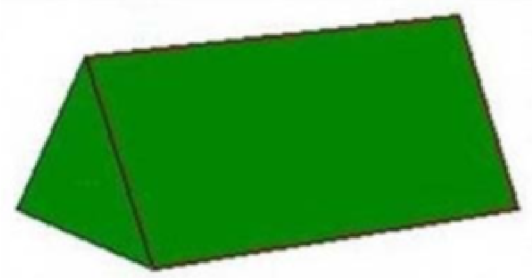
Pyramid: a polyhedron with a polygon base and triangular faces that meet at a point called the vertex (can be called an 'apex'). The pyramid is named according to the shape of its base.

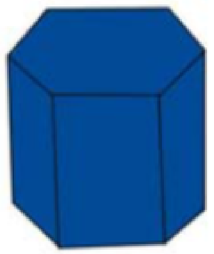




Types of solids

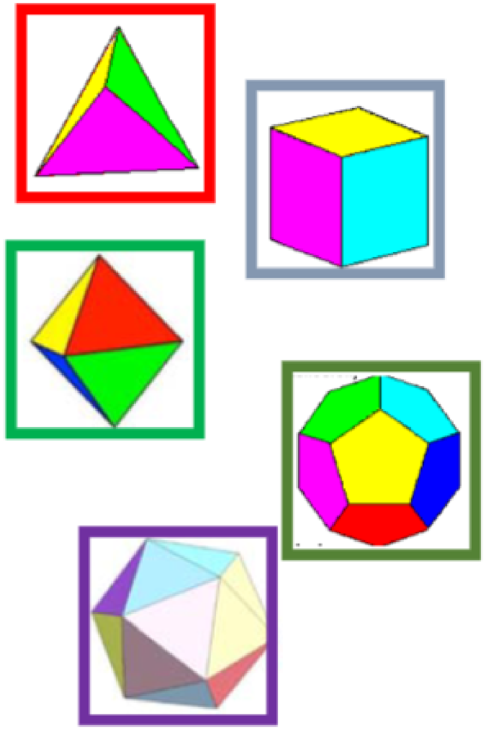
Prism: a polyhedron with parallel and congruent faces with the figure of the pair of congruent faces naming the prism and all of its remaining faces are parallelograms





Platonic Solids (regular polyhedra)

- Tetrahedron – 4 faces and all are equilateral triangles
- Cube (or hexahedron) – 6 faces and all are squares
- Octahedron – 8 faces and all are equilateral triangles
- Dodecahedron – 12 faces all that are regular pentagons
- Icosahedron – 20 faces all that are equilateral triangles

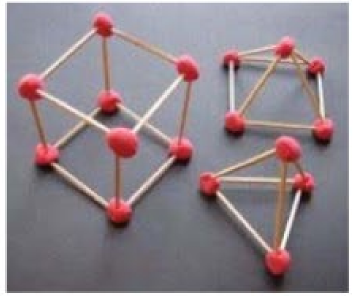


Representing with concrete materials

There are different ways that children can represent 3D solids through the use of concrete materials

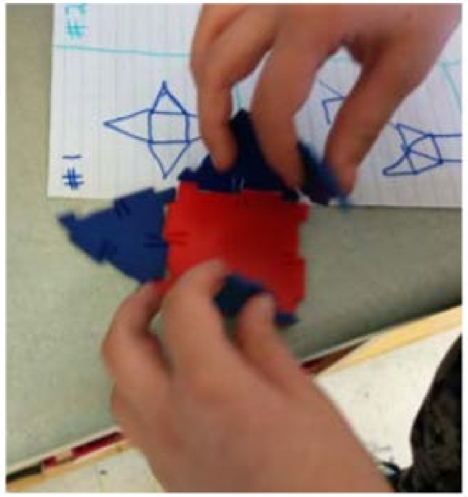
Polydron

Geo-Fix



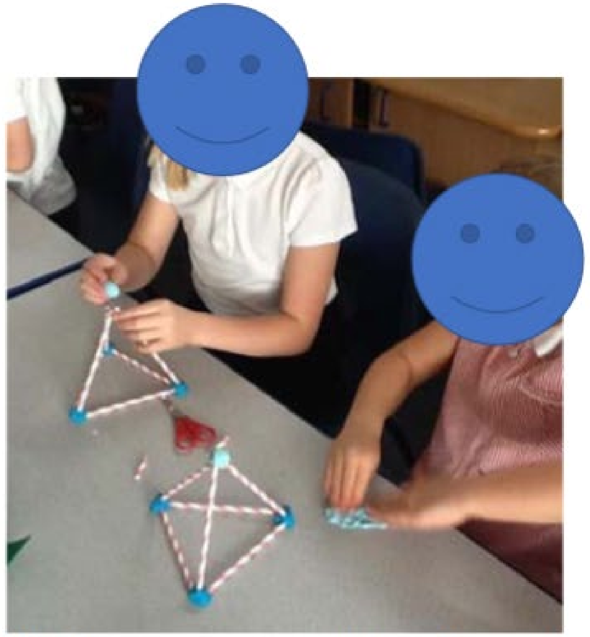
match sticks and plasticine

Representing 3D solids



Polydron represent the faces, edges, and vertices.

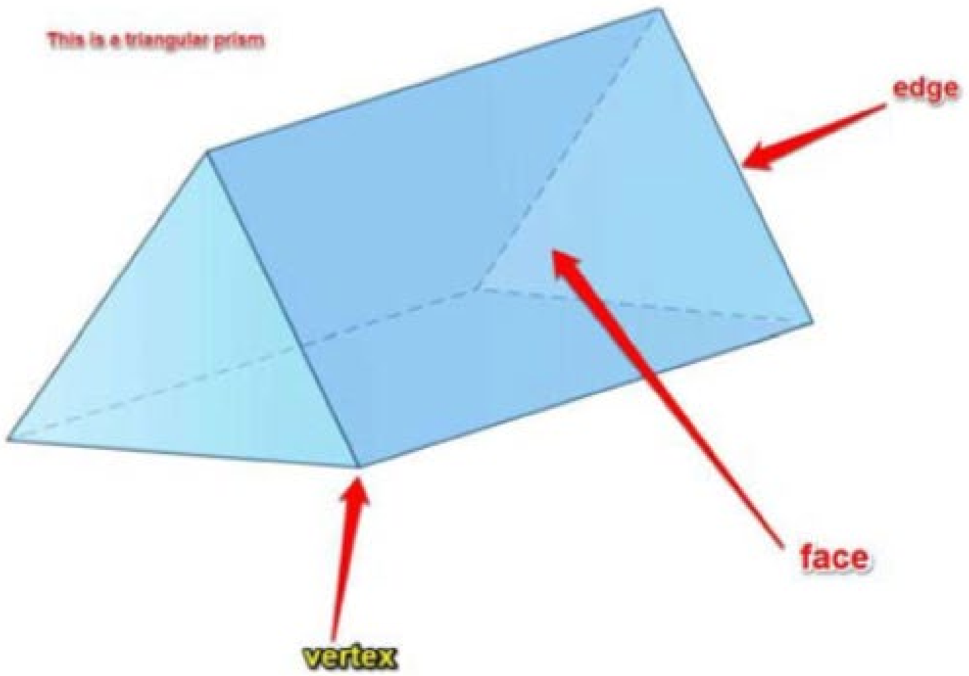
Magnetic Polydron can help children see the figure of the face



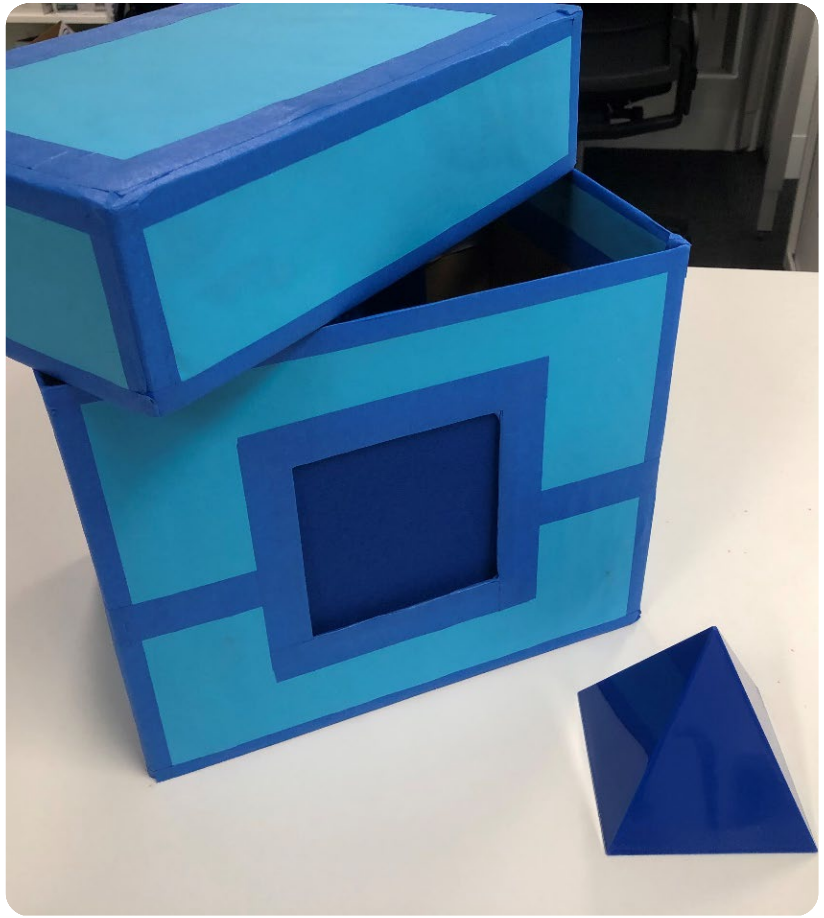
With these representations, the edges and vertices are seen clearly. Faces of the tetrahedron rely on visualisations.

Some types of 3D properties

- Edges
- Faces
- Vertices
- Angles
- Diagonals
- Symmetry
- Concavity

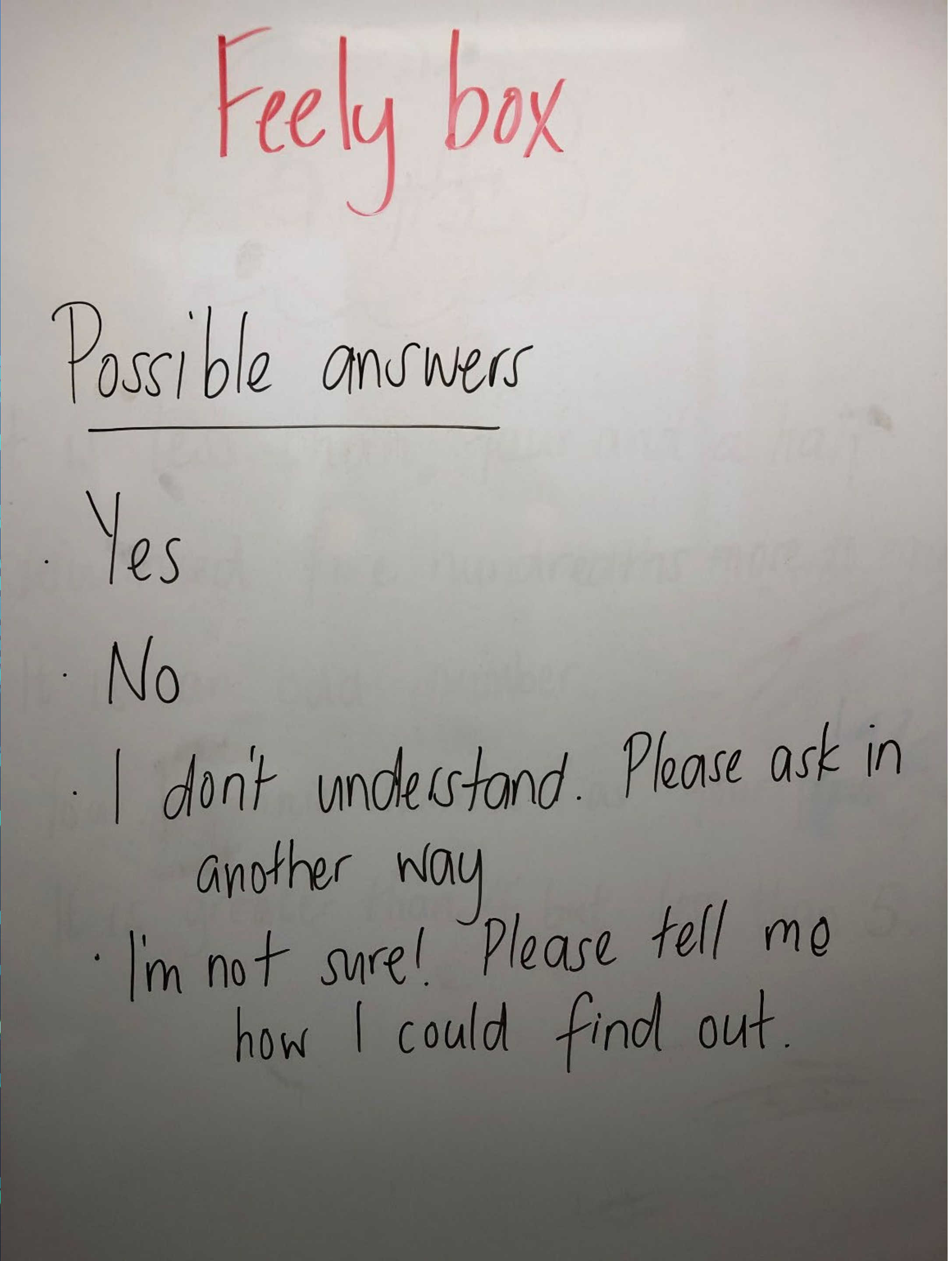


A Grade 3 child's labelling of properties of a triangular prism using the app, Skitch

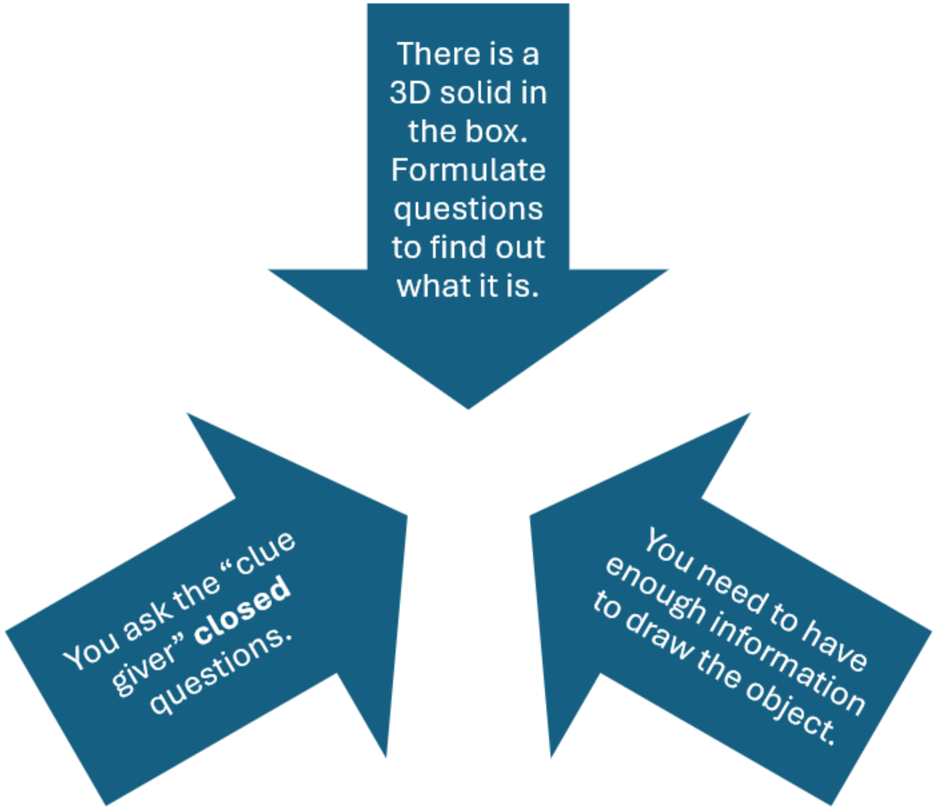


Feely box

- Put a 3D object into the box without students seeing except the feeler
- Choose 2 students
- One is the feeler
- One is the guider
- Guider can check in the box when they need to.


Feely box
Possible answers
· Yes
· No
· I don't understand. Please ask in another way
· I'm not sure! Please tell me how I could find out.

Feely Box


There is a 3D solid in the box. Formulate questions to find out what it is.
You ask the "clue giver" **closed** questions.
You need to have enough information to draw the object.

I don't understand the question. Please tell me in another way.

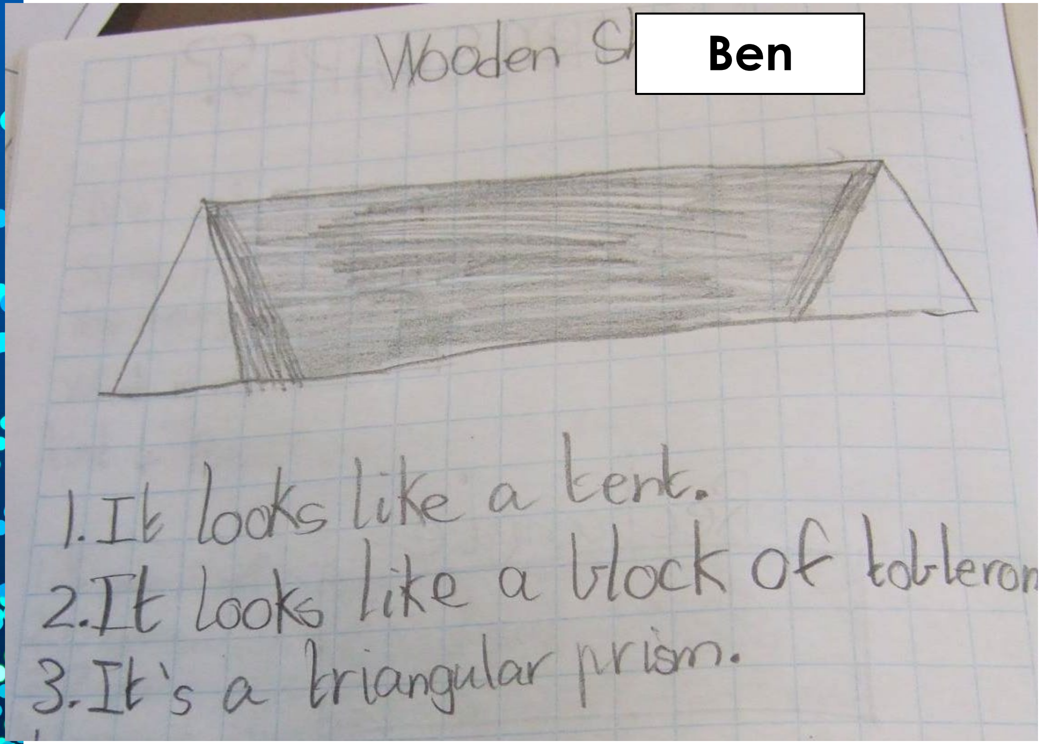
Launch

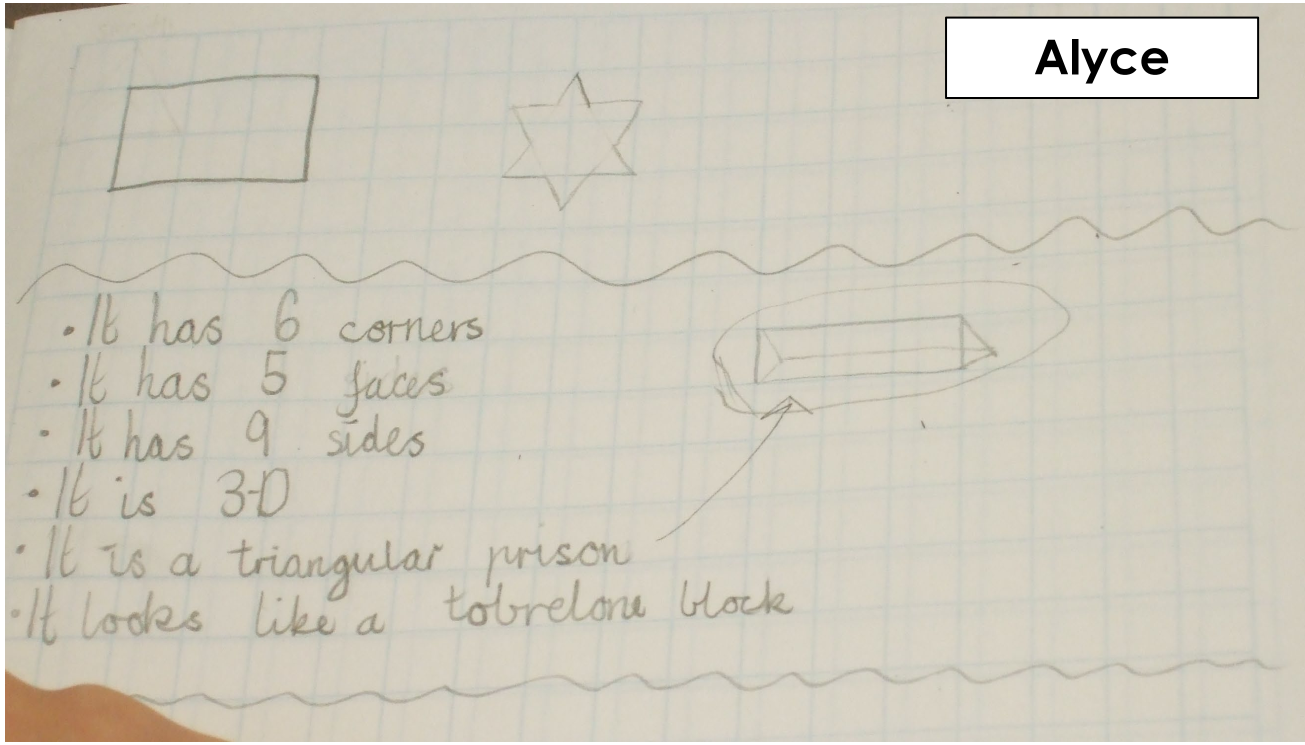
- Using Feely Box with children sitting on floor
- Inviting volunteers to lead the task
- Recording responses from students on IWB

Feely box clues

Launch

Work sample analysis



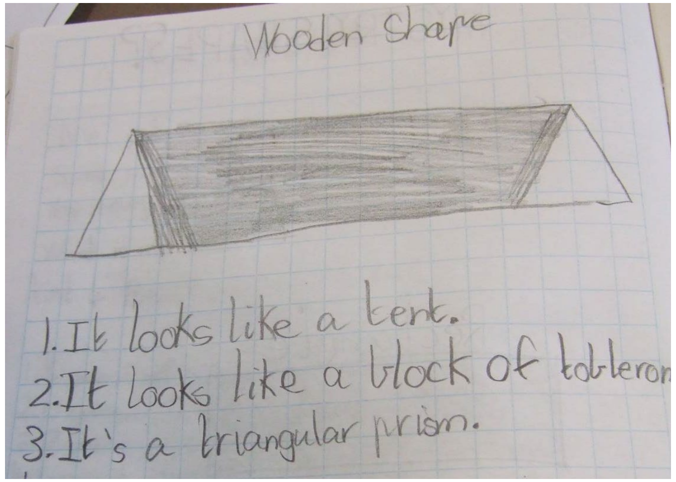


What feedback would you give to each student?

What's worked well…(WWW)

Even better if…(EBI)

Ben’s work sample



Van Hiele level:

Visualisation

Reason:

Ben is attending to the visual information; “looks like a tent” and “looks like a Toblerone”.

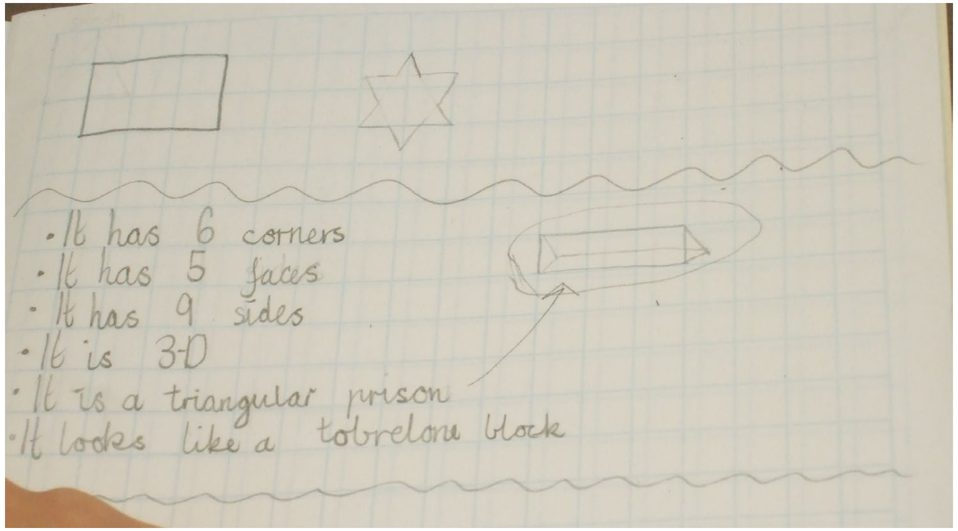
Next level:

Analysis

Possible teacher and learner actions:

- Bringing attention to properties (features) through questioning and learning experiences
- Exploring examples and non-examples
- Representing prisms through language and visual forms
- Building up technical-mathematical vocabulary

Alyce’s work sample



Van Hiele level:

Descriptive / Analytical

Reason:

Alyce is paying attention to properties

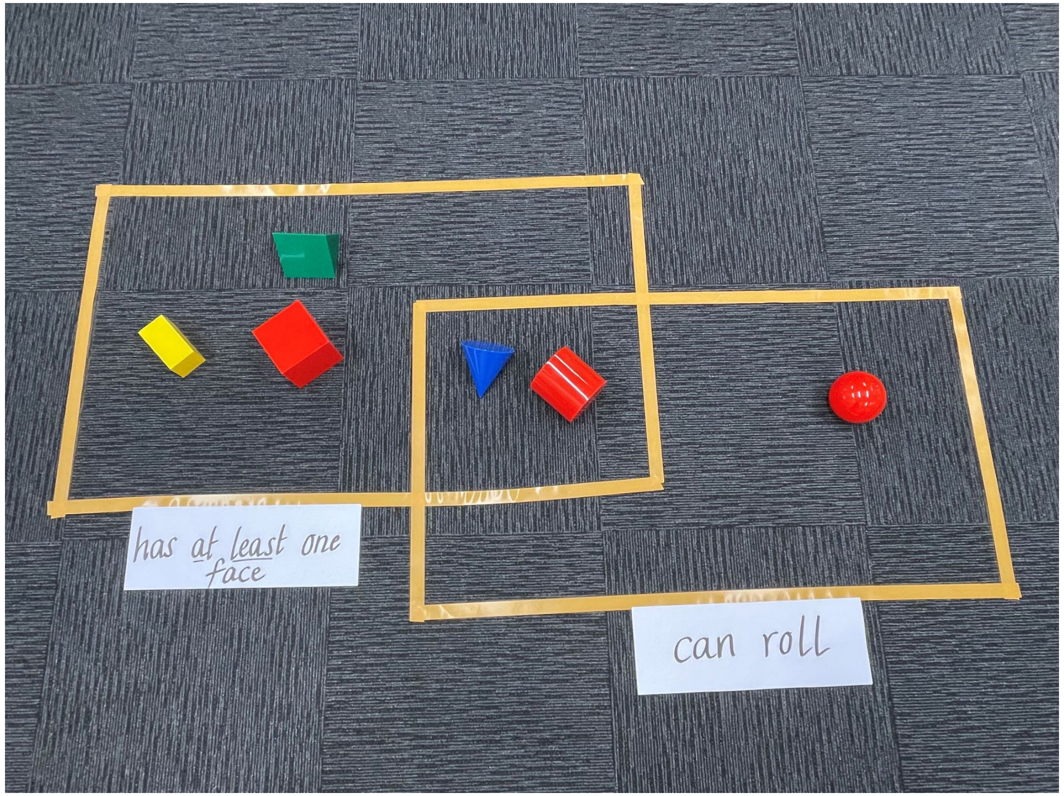
Next level:

Relational / Abstract

Possible teacher and learner actions:

- Connecting everyday terms with technical-mathematical vocabulary
- Comparing triangular prisms and pyramids
- Exploring similarities between triangular and rectangular prisms
- Investigating cross-sections of prisms

Classifying solids from the Feely Box



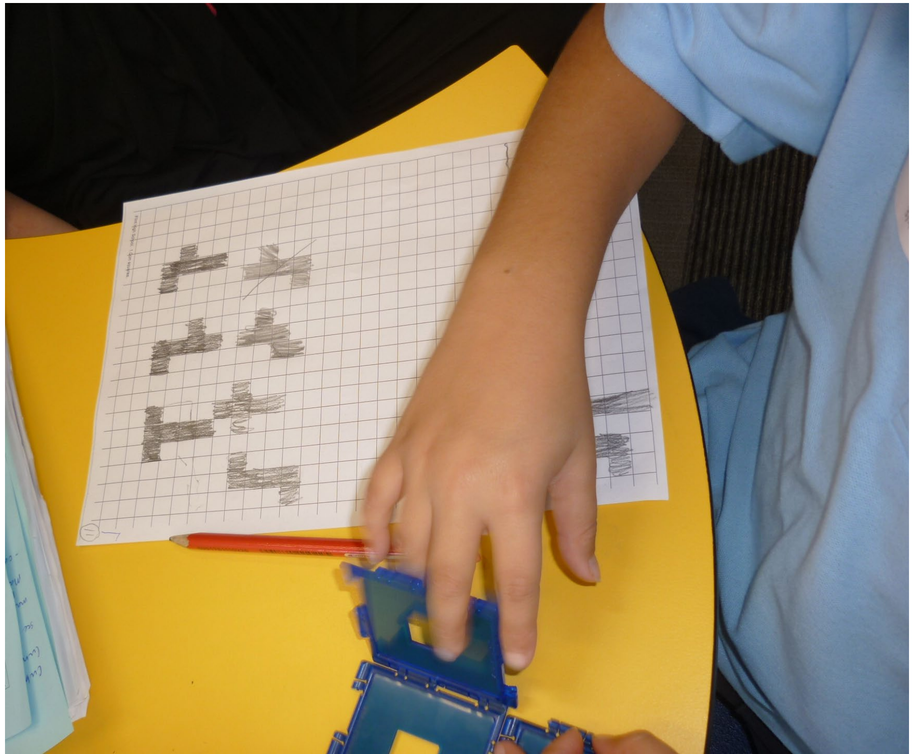
The teacher used the Feely Box over a series of days.

Solids that were used each time we placed on the ledge.

The teacher then asked the children how they might classify them once a collection of solids had been built up.

How many faces does a sphere have?

General consensus is that a sphere has no (0) face but rather one (1) **surface**



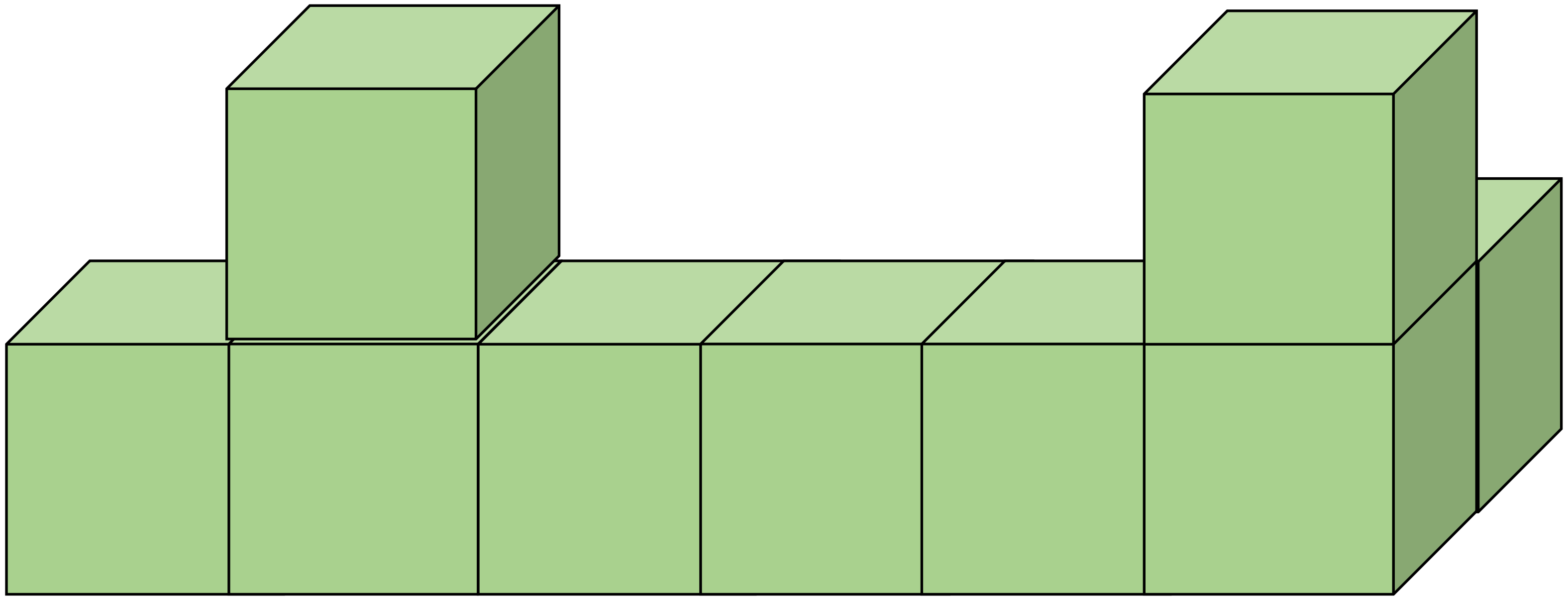
Exploring nets of the "Feely Box" solid

- Working systematically to find all nets for a cube
- Using Polydron and grid paper to record all possibilities
- Checking for duplicates and putting together convincing arguments

Look at my solid carefully.

I’m going to hide it from you now.

Can you duplicate the solid?
Convince me you have made its twin.



Look carefully.

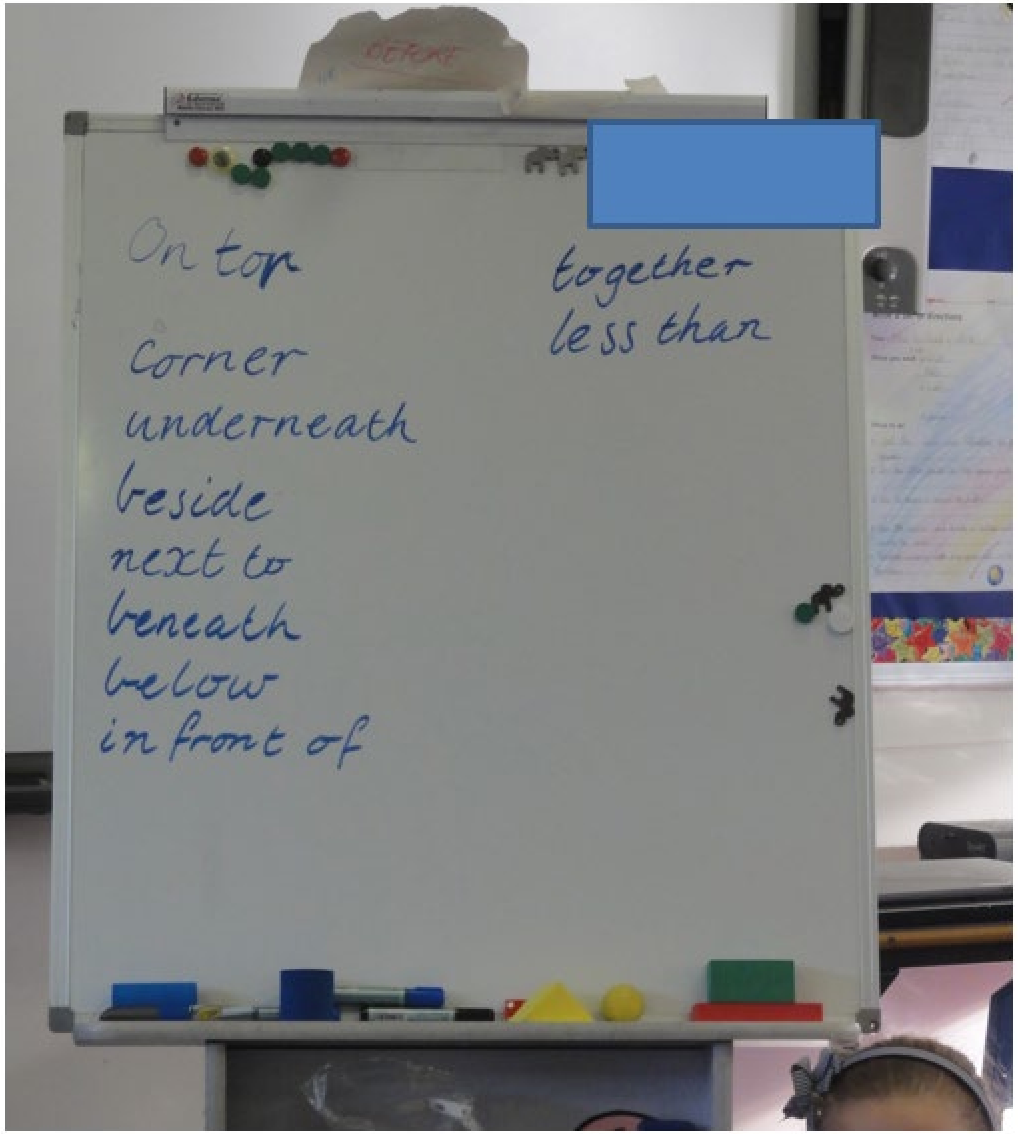
Launch

Take a photo of it,
like a smart phone,
using your mind's eye.

Explore

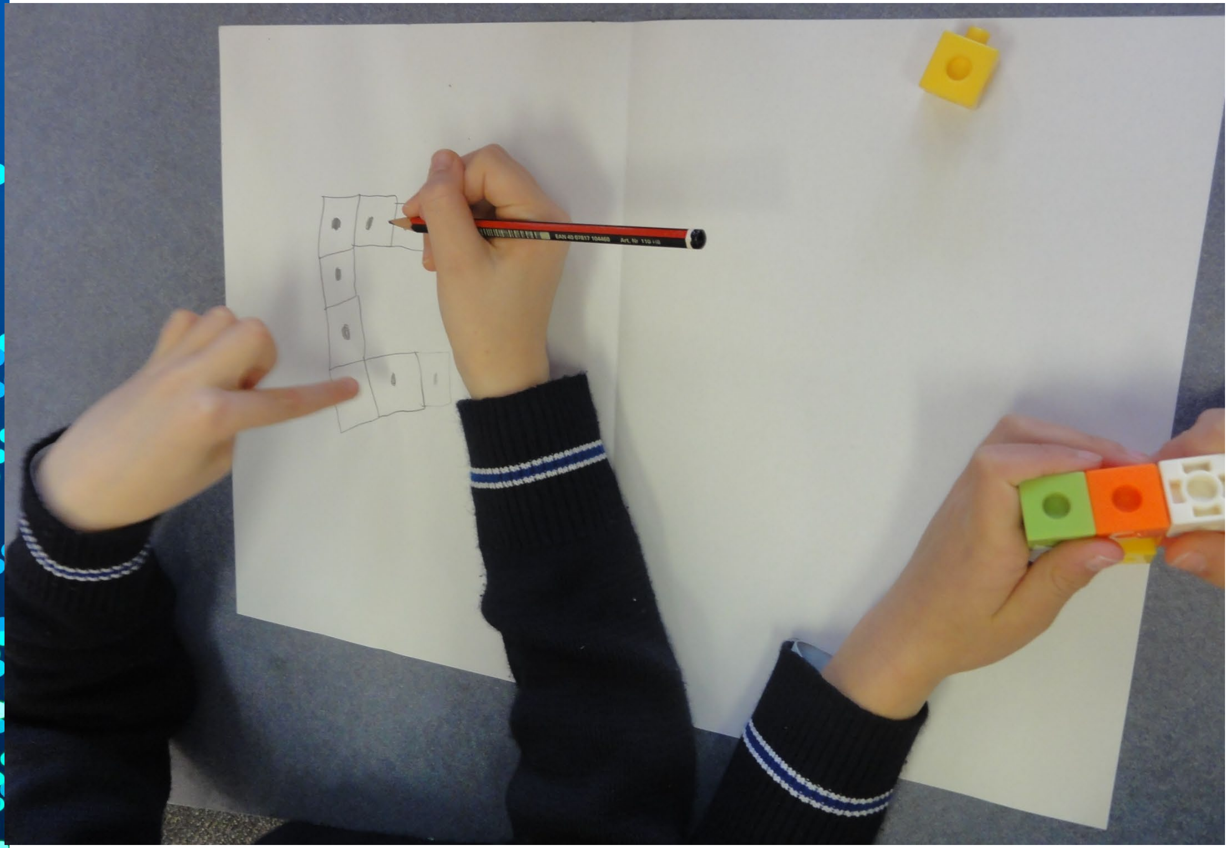
- Recreating the 3D solid using visualisations and concrete materials
- Using connecting cubes (already prepared on desks)
- Choosing to work individually or with a ‘learning partner’

Explore

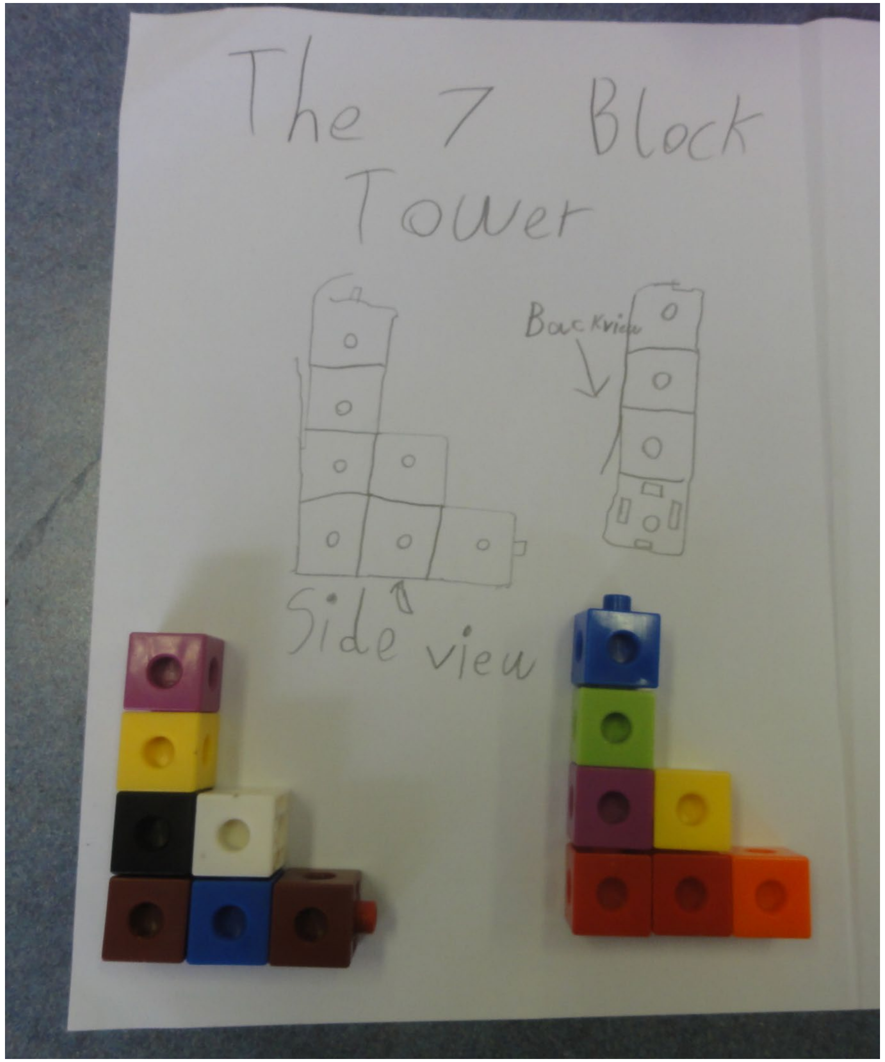


- Drawing out mathematical vocabulary after experiences of mathematical activity
- Making mathematical vocabulary visible to students
- Recording language to support next part of lesson: students' recording

Explore



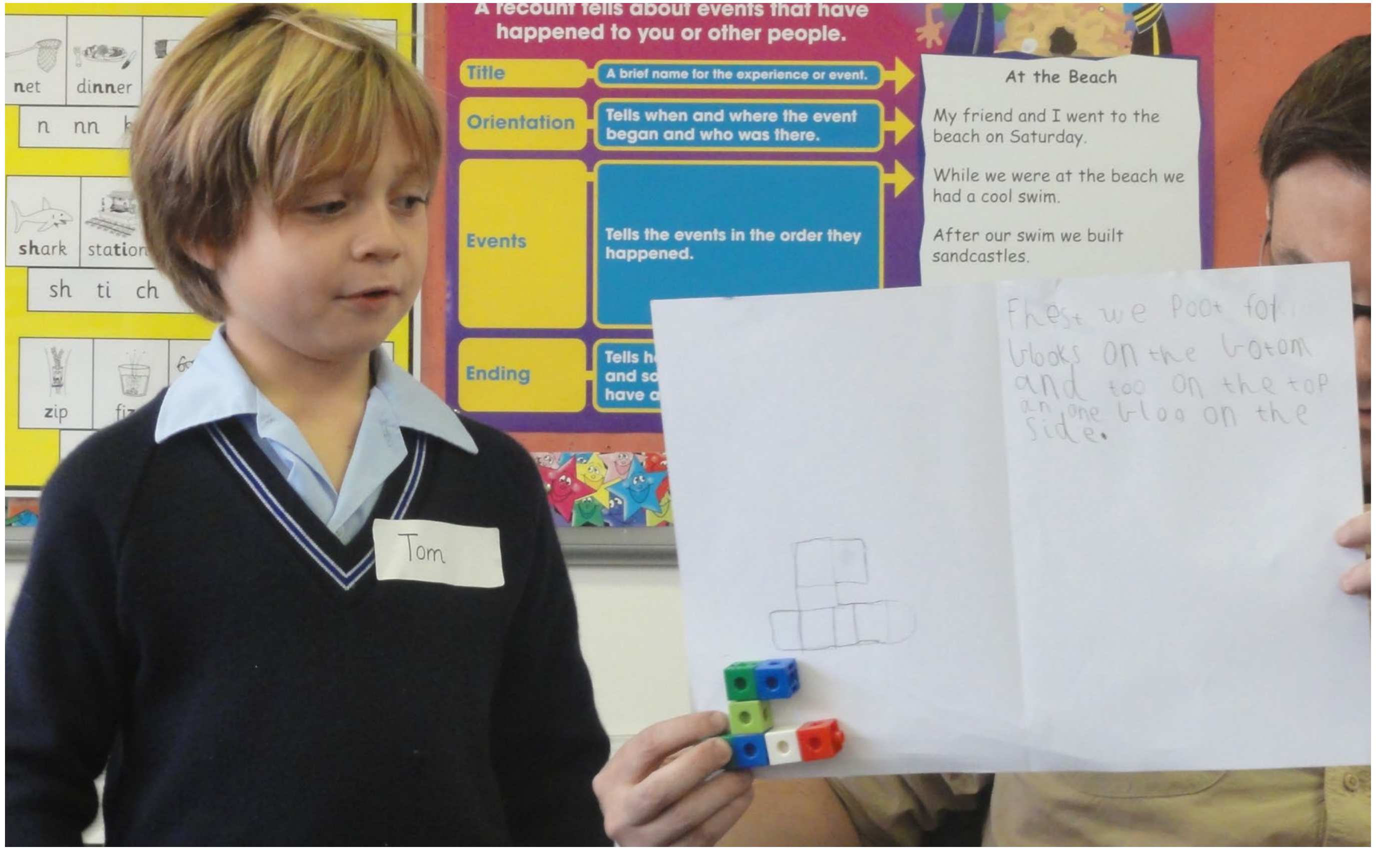
- Recording plans for another person to recreate a 'building'
- Using drawings and language to communicate mathematical thinking
- Linking 3D concrete material to 2D representations



Using the **Concrete-Representation-Abstract** (CRA) approach

- **Concrete** – creating structures using concrete materials (connecting cubes)
- **Representation** – using images/drawings that match 2D representation with 3D solid/object
- **Abstract** – using visualisations (mental images), symbols, and language to represent 3D solid/object

Summarise



- Orchestrating the discussion but carefully selecting children to share their reasoning and work sample (not all children share)
- Drawing out the mathematics focus

Extending to Years 4-8

Given this **orthogonal** view, what would this structure look like in 3D form?

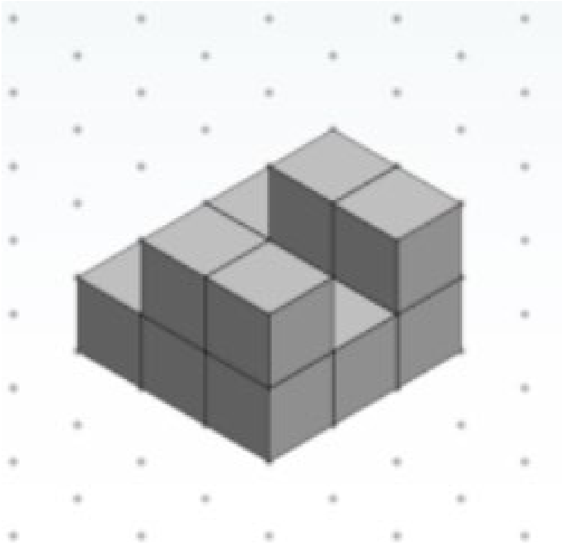
In what other ways could you represent this structure using 2D representations?

With a partner:

Partner 1: Create a 3D shape with the unifix blocks

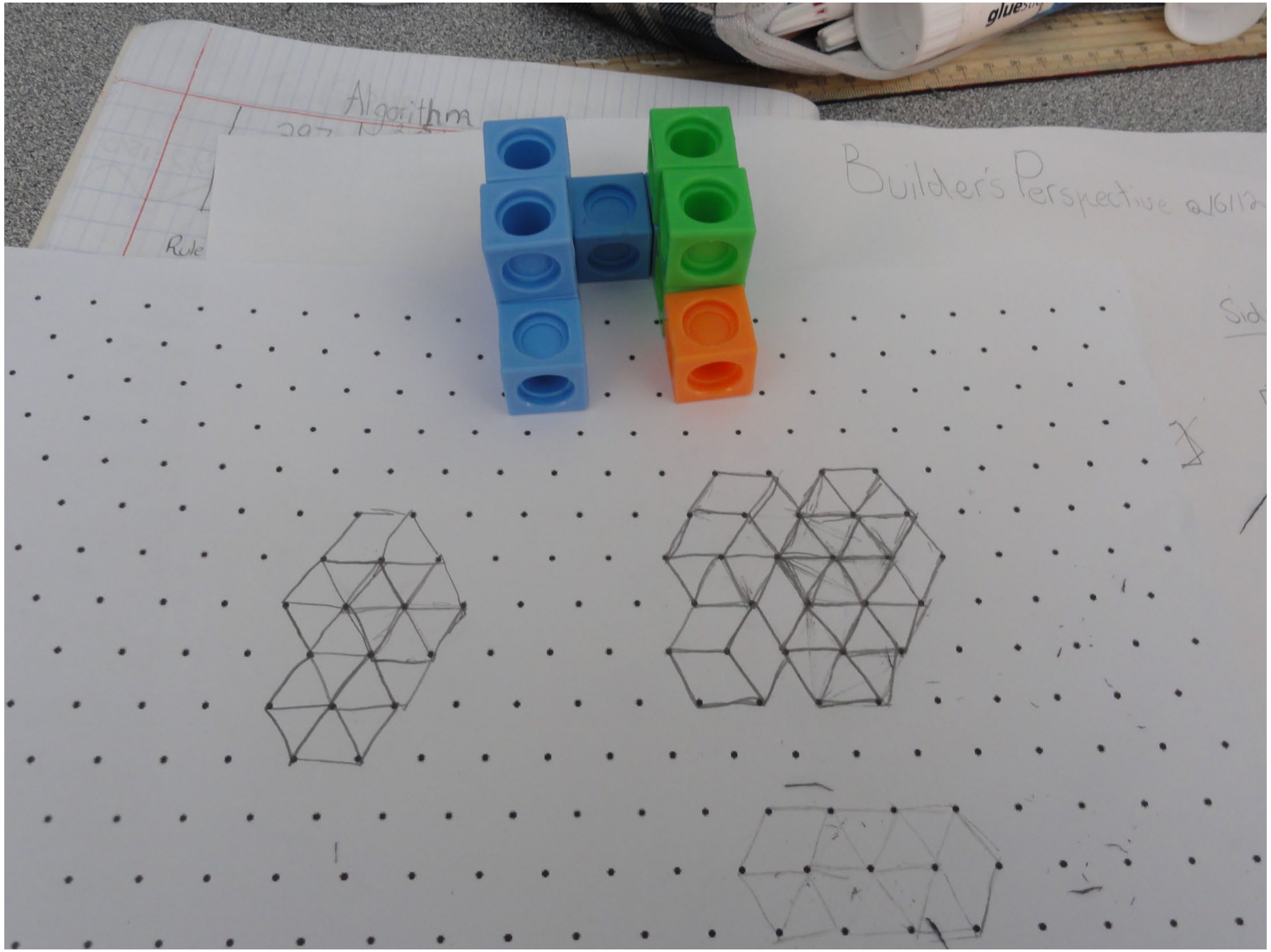
Partner 2: Use the NCTM Isometric tool to draw an orthogonal view of it.

Then swap roles.



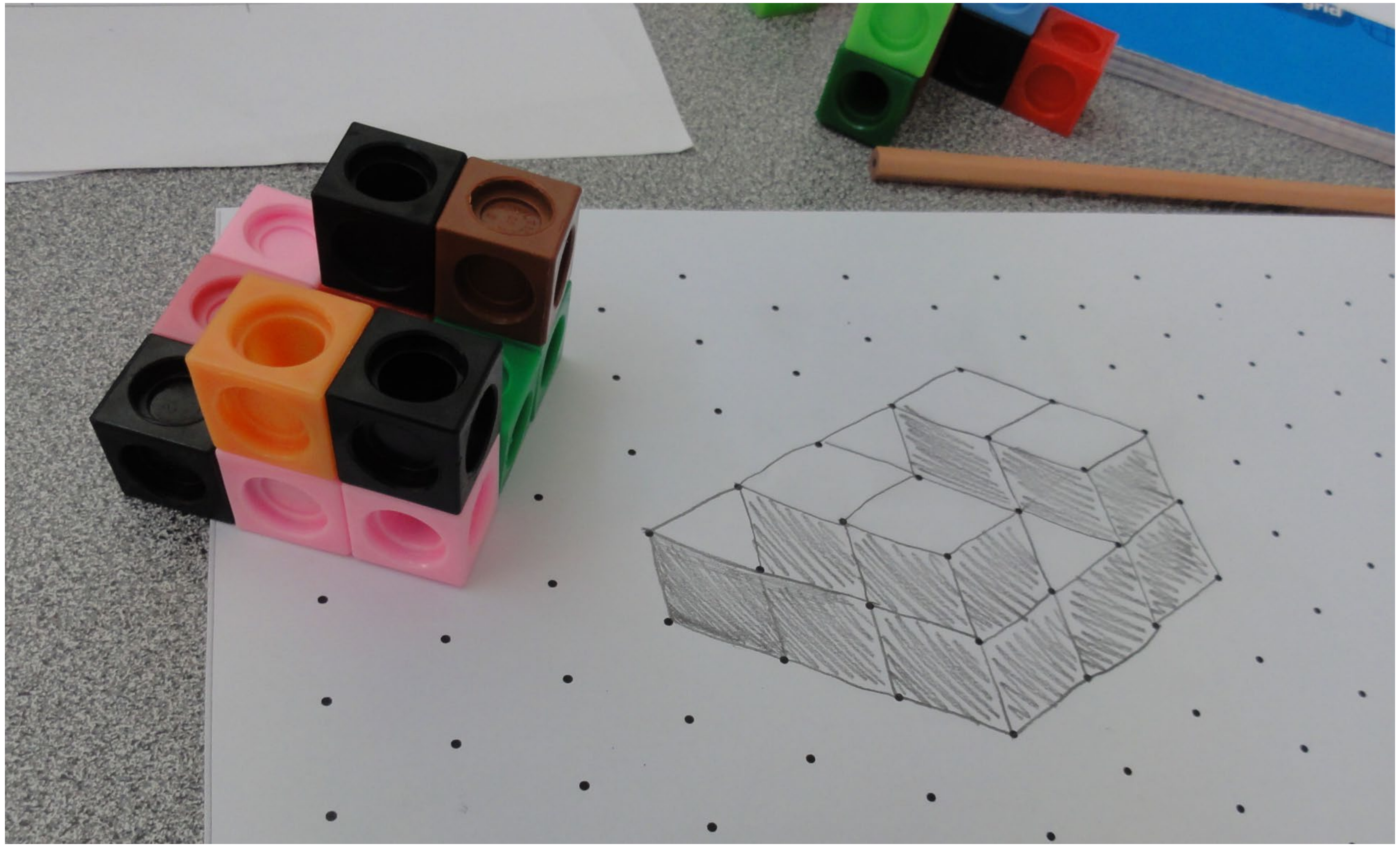
NCTM Isometric Drawing Tool

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Isometric-Drawing-Tool/

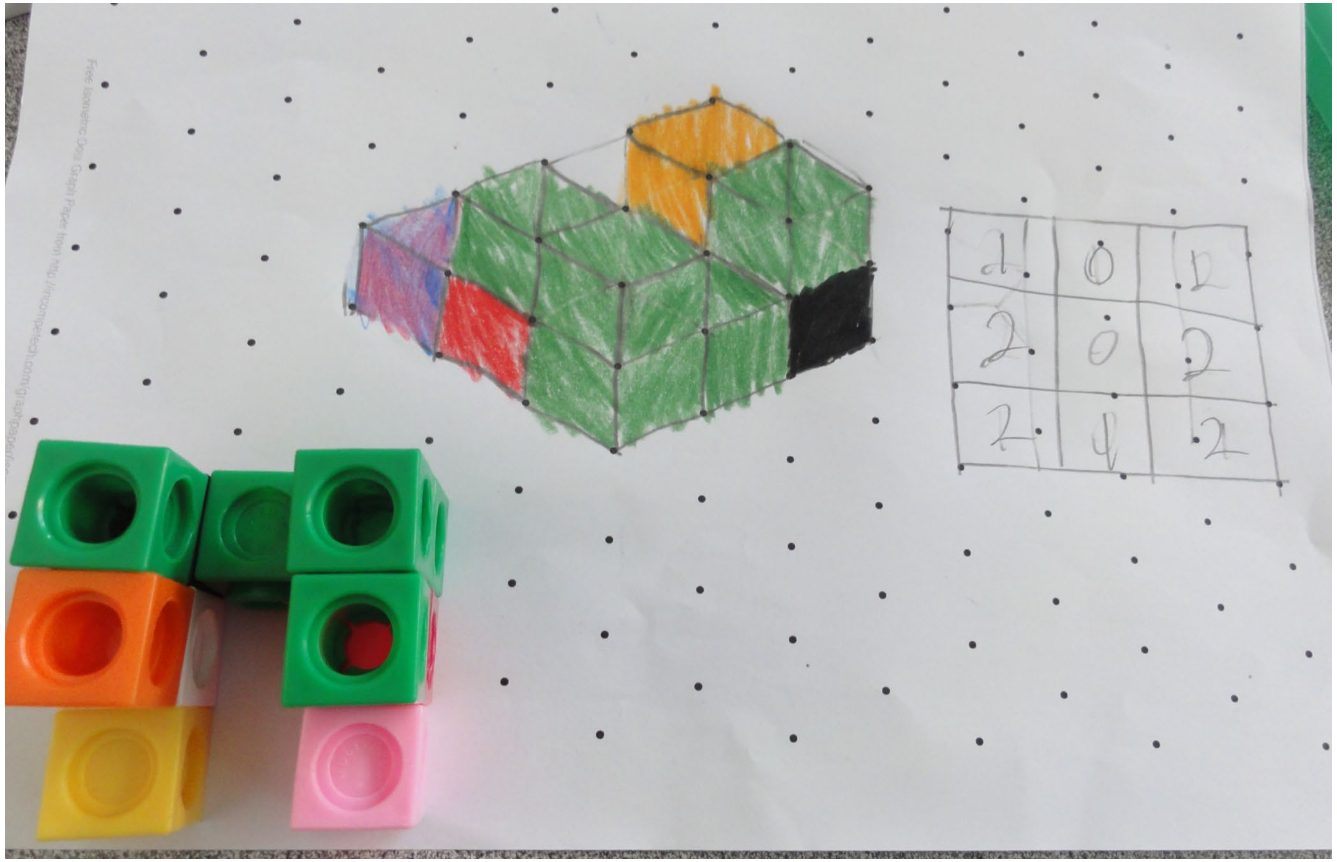


Representing 3D objects using isometric paper is challenging

Students need many experiences to become fluent with its use



This student decided to use shading to make further sense of her representation

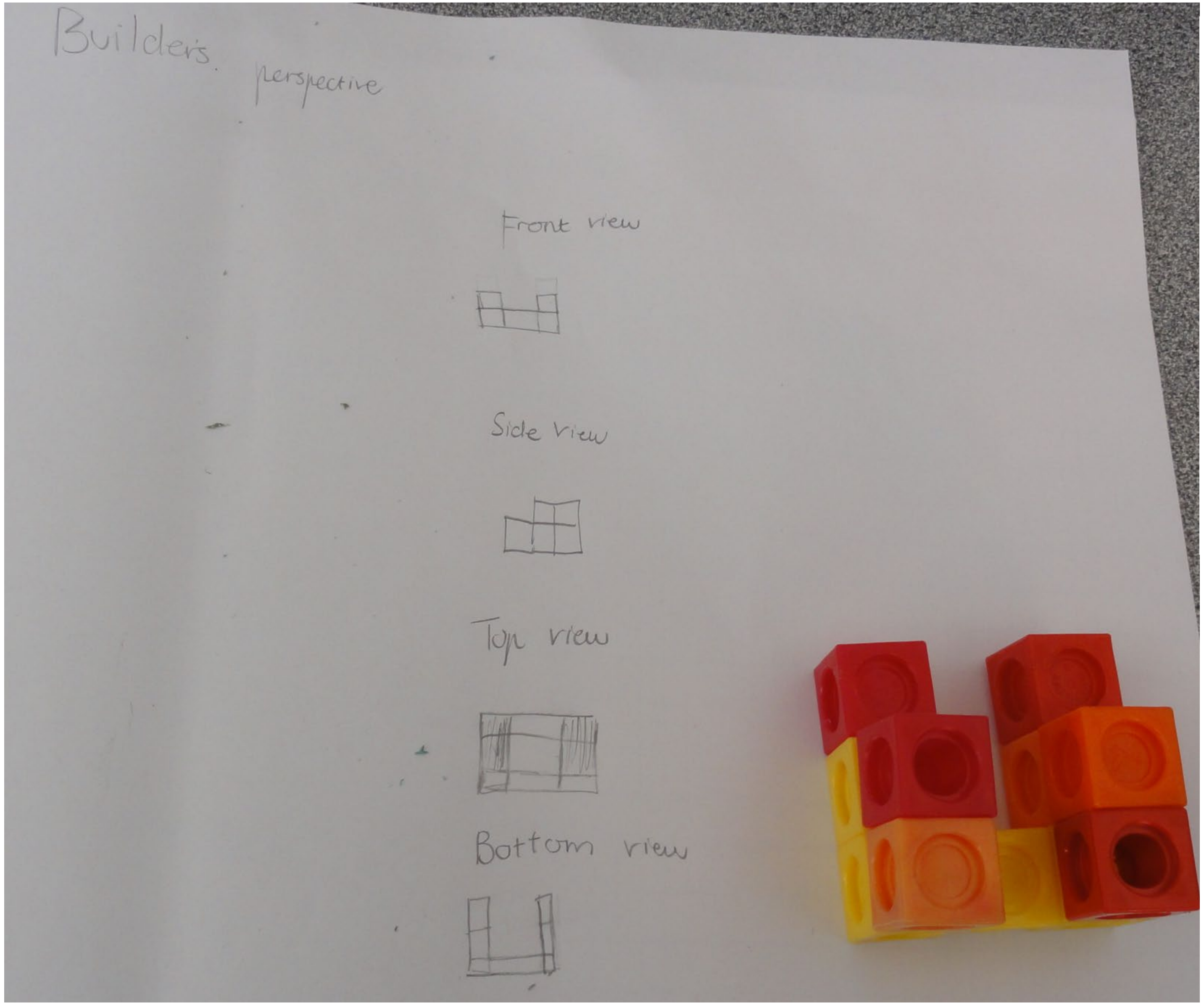


This student used three different representations:

Concrete – using connecting cubes

Representation - isometric drawing

Abstract – mat plan (symbols)



This student decided to create different views for the same structure (front view, side view, top view, etc.)

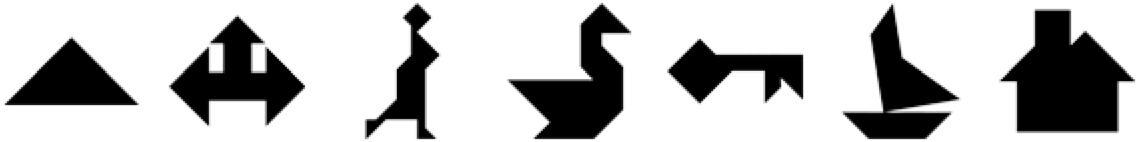
These are called "**orthogonal drawings**".

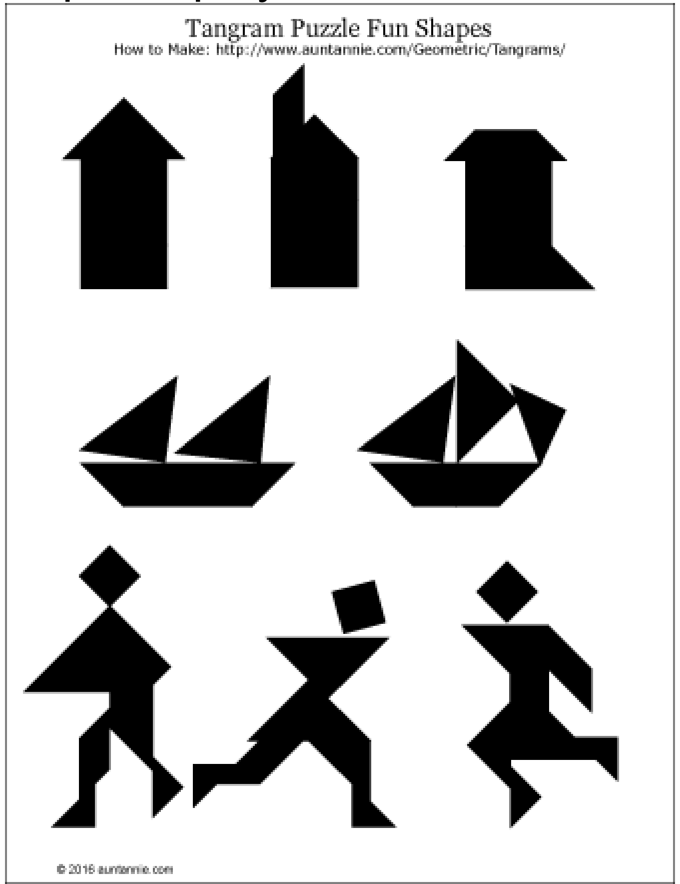
What is a Tangram?

A Tangram puzzle focuses on the objective to rearrange the seven separate pieces into a complete image of various sha

Coming from China, this logic game brings the mathematical thought of Asia and incorporates
it with bright colours, and charming figures.

Tangrams are an excellent way to increase mental ability, and are simple to play and suitable for all ages





- Use your set of Tangram pieces to create one of these images.
- What year levels do you think would enjoy these?

Resources

Van de Walle et al. (2019). Chapter 20 Geometric thinking and geometric concepts: Shapes and Properties (3D)

Mason, M (2009). The van Hiele levels of geometric thinkingLinks to an external site.

Recommended reading

Zimmermann, L., Foster, L, Michnick Golinkoff, & Hirsh-Pasek, K. (2019). Spatial thinking and STEM: How playing with blocks supports early math.Links to an external site. American Educator, 42(4), 22–27.

Event App

App Download Instructions

Step 1: Download the App 'Arinex One' from the App Store or Google Play

App Store

Google Play

Step 2: Enter Event Code: **mav**

Step 3: Enter the email you registered with

Step 4: Enter the Passcode you receive via email and click 'Verify'. Please be sure to check your Junk Mail for the email, or see the Registration Desk if you require further assistance.

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Be in it to WIN!